Asymptotic Statistics

This book is an introduction to the field of asymptotic statistics. The treatment is both practical and mathematically rigorous. In addition to most of the standard topics of an asymptotics course, including likelihood inference, M-estimation, the theory of asymptotic efficiency, U-statistics, and rank procedures, the book also presents recent research topics such as semiparametric models, the bootstrap, and empirical processes and their applications. The topics are organized from the central idea of approximation by limit experiments, which gives the book one of its unifying themes. This entails mainly the local approximation of the classical i.i.d. set up with smooth parameters by location experiments involving a single, normally distributed observation. Thus, even the standard subjects of asymptotic statistics are presented in a novel way. Suitable as a graduate or Master's level statistics text, this book will also give researchers an overview of research in asymptotic statistics.

One of the aims of the conference on which this book is based, was to provide a platform for the exchange of recent findings and new ideas inspired by the so-called Hungarian construction and other approximate methodologies. This volume of 55 papers is dedicated to Miklós Csörgö, a co-founder of the Hungarian construction school by the invited speakers and contributors to ICAMPS'97. This excellent treatise reflects the many developments in this field, while pointing to new directions to be explored. An unequalled contribution to research in probability and statistics.

This is the second edition of a coherent introduction to the subject of asymptotic statistics as it has developed over the past 50 years. It differs from the first edition in that it is now more 'reader friendly' and also includes a new chapter on Gaussian and Poisson experiments, reflecting their growing role in the field. Most of the subsequent chapters have been entirely rewritten and the nonparametrics of Chapter 7 have been amplified. The volume is not intended to replace monographs on specialized subjects, but will help to place them in a coherent perspective. It thus represents a link between traditional material - such as maximum likelihood, and Wald's Theory of Statistical Decision Functions -- together with comparison and distances for experiments. Much of the material has been taught in a second year graduate course at Berkeley for 30 years.

Asymptotic Statistics
Cambridge University Press

The primary aim of this book is to provide modern statistical techniques and theory for stochastic processes. The stochastic processes mentioned here are not restricted to the usual AR, MA, and ARMA processes. A wide variety of stochastic processes, including non-Gaussian linear processes, long-memory processes, nonlinear processes, non-ergodic processes and diffusion processes are described. The authors discuss estimation and testing theory and many other relevant statistical methods and techniques.

Bayesian nonparametrics comes of age with this landmark text synthesizing theory, methodology and computation. Asymptotic methods provide important tools for approximating and analysing functions that arise in probability and statistics. Moreover, the conclusions of asymptotic analysis often supplement the conclusions obtained by numerical methods. Providing a broad toolkit of analytical methods, Expansions and Asymptotics for Statistics shows how asymptotics, when coupled with numerical methods, becomes a powerful way to acquire a deeper understanding of the techniques used in probability and statistics. The book first discusses the role of expansions and asymptotics in statistics, the basic properties of power series and asymptotic series, and the study of rational approximations to
functions. With a focus on asymptotic normality and asymptotic efficiency of standard estimators, it covers various applications, such as the use of the delta method for bias reduction, variance stabilisation, and the construction of normalising transformations, as well as the standard theory derived from the work of R.A. Fisher, H. Cramér, L. Le Cam, and others. The book then examines the close connection between saddle-point approximation and the Laplace method. The final chapter explores series convergence and the acceleration of that convergence.

1 To the king, my lord, from your servant Balasi: 2 ... The king should have a look. Maybe the scribe who reads to the king did not understand... shall I personally show, with this tablet that I am sending to the king, my lord, how the omen was written. 3 Really, he who has not followed the text with his finger cannot possibly understand it. This book is about optimally robust functionals and their unbiased estimators and tests. Functionals extend the parameter of the assumed ideal center model to neighborhoods of this model that contain the actual distribution. The two principal questions are (F): Which functional to choose? and (P): Which statistical procedure to use for the selected functional? Using a local asymptotic framework, we deal with both problems by linking up nonparametric statistical optimality with infinitesimal robustness criteria. Thus, seemingly separate developments in robust statistics are presented in a unifying way.

This volume provides an exposition of some fundamental aspects of the asymptotic theory of statistical experiments. The most important of them is “how to construct asymptotically optimal decisions if we know the structure of optimal decisions for the limit experiment”.


Readership: Researchers in probability and statistics.

Keywords: Comparison of Statistical Experiments; Mixed Local Asymptotic Normality; Convergence of Experiments; Likelihood Ratio Processes; Contiguity; Autoregressive Models; Minimax Bound; Local Asymptotic Normality

Reviews: “It is an interesting, welcome addition to the literature, and it contains many new insights. I congratulate the authors for writing this comprehensive monograph on a difficult subject.” Mathematical Reviews “The book is a highlight in modern mathematical statistics which offers a lot of new concepts. It recalls the brilliant methodology of Le Cam's Theory and the first chapters may be used as introduction into this field.” Mathematics Abstracts

Traditions of the 150-year-old St. Petersburg School of Probability and Statistics had been developed by many prominent scientists including P. L. Chebychev, A. M. Lyapunov, A. A. Markov, S. N. Bernstein, and Yu. V. Linnik. In 1948, the Chair of Probability and Statistics was established at the Department of Mathematics and Mechanics of the St. Petersburg State University with Yu. V. Linik being its founder and also the first Chair. Nowadays, alumni of this Chair are spread around Russia, Lithuania, France, Germany, Sweden, China, the United States, and Canada. The fiftieth anniversary of this Chair was celebrated by an International Conference, which was held in St. Petersburg from June 24-28, 1998. More than 125 probabilists and statisticians from 18 countries (Azerbaijan, Canada, Finland, France, Germany, Hungary, Israel, Italy, Lithuania, The Netherlands, Norway, Poland, Russia, Taiwan, Turkey, Ukraine, Uzbekistan, and the United States) participated in this International Conference in order to discuss the current state and perspectives of Probability and Mathematical Statistics. The conference was organized jointly by St. Petersburg State University, St. Petersburg branch of Mathematical Institute, and the Euler Institute, and was partially sponsored by the Russian Foundation of Basic Researches. The main theme of the Conference was chosen in the tradition of the St.

Here is a practical and mathematically rigorous introduction to the field of asymptotic statistics. In addition to most of the standard topics of
an asymptotics course—likelihood inference, M-estimation, the theory of asymptotic efficiency, U-statistics, and rank procedures—the book also presents recent research topics such as semiparametric models, the bootstrap, and empirical processes and their applications. The topics are organized from the central idea of approximation by limit experiments, one of the book's unifying themes that mainly entails the local approximation of the classical i.i.d. set up with smooth parameters by location experiments involving a single, normally distributed observation.

These notes are based on lectures presented during the seminar on "Asymptotic Statistics" held at SchloB Reisensburg, Gunzburg, May 29-June 5, 1988. They consist of two parts, the theory of asymptotic expansions in statistics and probabilistic aspects of the asymptotic distribution theory in nonparametric statistics. Our intention is to provide a comprehensive presentation of these two subjects, leading from elementary facts to the advanced theory and recent results. Prospects for further research are also included. We would like to thank all participants for their stimulating discussions and their interest in the subjects, which made lecturing very pleasant. Special thanks are due H. Zimmer for her excellent typing. We would also like to take this opportunity to express our thanks to the Gesellschaft fur mathematische Forschung and to the Deutsche Mathematiker Vereinigung, especially to Professor G. Fischer, for the opportunity to present these lectures and to the Birkhauser Verlag for the publication of these lecture notes. R. Bhattacharya, M. Denker

Part I: Asymptotic Expansions in Statistics

Rabi Bhattacharya

1. CRAMER-EDGEWORTH EXPANSIONS

Let Q be a probability measure on \((\mathbb{R}^k, \mathcal{B})\), where \(\mathcal{B}\) denotes the Borel sigmafield on \(\mathbb{R}^k\). Assume that the \(s\)-th absolute moment of Q is finite, \(\mathbb{E}[|X|^s] < \infty\). Define \(P_n := \sum_{i=1}^{n} |X_i|^s\).

A coherent introductory text from a groundbreaking researcher, focusing on clarity and motivation to build intuition and understanding. When certain parameters in the problem tend to limiting values (for example, when the sample size increases indefinitely, the intensity of the noise approaches zero, etc.) To address the problem of asymptotically optimal estimators consider the following important case. Let \(X_1, X_2, \ldots, X_n\) be independent observations with the joint probability density \(f(x, \theta)\) (with respect to the Lebesgue measure on the real line) which depends on the unknown parameter \(\theta \in \Theta \subseteq \mathbb{R}^p\). It is required to derive the best (asymptotically) estimator \(\hat{\theta}_n\) of \(\theta\) which is consistent and asymptotically normal. The first question which arises in connection with this problem is how to compare different estimators or, equivalently, how to assess their quality, in terms of the mean square deviation from the parameter or perhaps in some other way. The presently accepted approach to this problem, resulting from A. Wald's contributions, is as follows: introduce a nonnegative function \(w(\theta, \hat{\theta})\) (the loss function) and given two estimators \(\hat{\theta}_1\) and \(\hat{\theta}_2\) the estimator for which the expected loss (risk) \(E_w(\hat{\theta}_j, \theta)\), \(j = 1, 2\), is smallest is called the better with respect to \(W_n\) at point \(\theta\). Obviously, such a method of comparison is not without its defects.

This volume provides a modern introduction to stochastic geometry, random fields and spatial statistics at a (post)graduate level. It is focused on asymptotic methods in geometric probability including weak and strong limit theorems for random spatial structures (point processes, sets, graphs, fields) with applications to statistics. Written as a contributed volume of lecture notes, it will be useful not only for students but also for lecturers and researchers interested in geometric probability and related subjects. This book presents recent non-asymptotic results for approximations in multivariate statistical analysis. The book is unique in its focus on results with the correct error structure for all the parameters involved. Firstly, it discusses the computable error bounds on correlation coefficients, MANOVA tests and discriminant functions studied in recent papers. It then introduces new areas of
research in high-dimensional approximations for bootstrap procedures, Cornish–Fisher expansions, power-divergence statistics and approximations of statistics based on observations with random sample size. Lastly, it proposes a general approach for the construction of non-asymptotic bounds, providing relevant examples for several complicated statistics. It is a valuable resource for researchers with a basic understanding of multivariate statistics.

The series is devoted to the publication of high-level monographs and surveys which cover the whole spectrum of probability and statistics. The books of the series are addressed to both experts and advanced students. The use in statistical theory of approximate arguments based on such methods as local linearization (the delta method) and approximate normality has a long history. Such ideas play at least three roles. First they may give simple approximate answers to distributional problems where an exact solution is known in principle but difficult to implement. The second role is to yield higher-order expansions from which the accuracy of simple approximations may be assessed and where necessary improved. Thirdly the systematic development of a theoretical approach to statistical inference that will apply to quite general families of statistical models demands an asymptotic formulation, as far as possible one that will recover 'exact' results where these are available. The approximate arguments are developed by supposing that some defining quantity, often a sample size but more generally an amount of information, becomes large: it must be stressed that this is a technical device for generating approximations whose adequacy always needs assessing, rather than a 'physical' limiting notion. Of the three roles outlined above, the first two are quite close to the traditional roles of asymptotic expansions in applied mathematics and much of the very extensive literature on the asymptotic expansion of integrals and of the special functions of mathematical physics is quite directly relevant, although the recasting of these methods into a probability mould is quite often enlightening.

This unique book delivers an encyclopedic treatment of classic as well as contemporary large sample theory, dealing with both statistical problems and probabilistic issues and tools. The book is unique in its detailed coverage of fundamental topics. It is written in an extremely lucid style, with an emphasis on the conceptual discussion of the importance of a problem and the impact and relevance of the theorems. There is no other book in large sample theory that matches this book in coverage, exercises and examples, bibliography, and lucid conceptual discussion of issues and theorems.

A unified treatment that presents powerful new methods to evaluate explicitly different kinds of efficiencies. The series is devoted to the publication of monographs and high-level textbooks in mathematics, mathematical methods and their applications. Apart from covering important areas of current interest, a major aim is to make topics of an interdisciplinary nature accessible to the non-specialist. The works in this series are addressed to advanced students and researchers in mathematics and theoretical physics. In addition, it can serve as a guide for lectures and seminars on a graduate level. The series de Gruyter Studies in Mathematics was founded ca. 30 years ago by the late Professor Heinz Bauer and Professor Peter Gabriel with the aim to establish a series of monographs and textbooks of high standard, written by scholars with an international reputation presenting current fields of research in pure and applied mathematics. While the editorial board of the Studies has changed with the years,
the aspirations of the Studies are unchanged. In times of rapid growth of mathematical knowledge carefully written monographs and textbooks written by experts are needed more than ever, not least to pave the way for the next generation of mathematicians. In this sense the editorial board and the publisher of the Studies are devoted to continue the Studies as a service to the mathematical community. Please submit any book proposals to Niels Jacob.

In particular up-to-date information is presented in detection of systematic changes, in series of observation, in robust regression analysis, in numerical empirical processes and in related areas of actuarial sciences. This work presents a collection of 18 papers, many of which are surveys, on asymptotic theory in probability and statistics, with applications to a wide variety of problems. This volume comprises three parts: limit theorems, statistics and applications, and mathematical finance and insurance. It is intended for graduate students in probability and statistics, and for researchers in related areas.

Differential geometry provides an aesthetically appealing and often-revealing view of statistical inference. Beginning with an elementary treatment of one-parameter statistical models and ending with an overview of recent developments, this is the first book to provide an introduction to the subject that is largely accessible to readers not already familiar with differential geometry. It also gives a streamlined entry into the field to readers with richer mathematical backgrounds. Much space is devoted to curved exponential families, which are of interest not only because they may be studied geometrically but also because they are analytically convenient, so that results may be derived rigorously. In addition, several appendices provide useful mathematical material on basic concepts in differential geometry. Topics covered include the following: * Basic properties of curved exponential families * Elements of second-order, asymptotic theory * The Fisher-Efron-Amari theory of information loss and recovery * Jeffreys-Rao information-metric Riemannian geometry * Curvature measures of nonlinearity * Geometrically motivated diagnostics for exponential family regression * Geometrical theory of divergence functions * A classification of and introduction to additional work in the field

0.1. The aim of the book Our "Contributions to a General Asymptotic Statistical Theory" (Springer Lecture Notes in Statistics, Vol. 13, 1982, called "Vol. I" in the following) suggest to describe the local structure of a general family \( \sim \) of probability measures by its tangent space, and the local behavior of a functional \( K: \sim \sim k \) by its gradient. Starting from these basic concepts, asymptotic envelope power functions for tests and asymptotic bounds for the concentration of estimators are obtained, and heuristic procedures are suggested for the construction of test and estimator-sequences attaining these bounds. In the present volume, these asymptotic investigations are carried one step further: From approximations by limit distributions to approximations by Edgeworth expansions, 1 2 adding one term (of order \( n^{-1/2} \)) to the limit distribution. As in Vol. I, the investigation is "general" in the sense of dealing with arbitrary families of probability measures and arbitrary functionals. The investigation is special in the sense that it is restricted to statistical procedures based on independent, identically distributed observations. 2 Moreover, it is special in the sense that its concern are "regular" models (i.e. families of probability measures and functionals which are subject to certain general conditions, like differentiability). Irregular models are certainly of mathematical interest. Since they are hardly of any practical relevance, it appears justifiable to exclude them at this stage of the investigation.

This textbook is devoted to the general asymptotic theory of statistical experiments. Local asymptotics for statistical models in the sense of local asymptotic (mixed) normality or local asymptotic quadraticity make up the core of the book. Numerous examples deal with classical independent and identically distributed models and with stochastic processes. The book can be read in different ways, according to possibly
different mathematical preferences of the reader. One reader may focus on the statistical theory, and thus on the chapters about Gaussian shift models, mixed normal and quadratic models, and on local asymptotics where the limit model is a Gaussian shift or a mixed normal or a quadratic experiment (LAN, LAMN, LAQ). Another reader may prefer an introduction to stochastic process models where given statistical results apply, and thus concentrate on subsections or chapters on likelihood ratio processes and some diffusion type models where LAN, LAMN or LAQ occurs. Finally, readers might put together both aspects. The book is suitable for graduate students starting to work in statistics of stochastic processes, as well as for researchers interested in a precise introduction to this area. This book is designed to bridge the gap between traditional textbooks in statistics and more advanced books that include the sophisticated nonparametric techniques. It covers topics in parametric and nonparametric large-sample estimation theory. The exposition is based on a collection of relatively simple statistical models. It gives a thorough mathematical analysis for each of them with all the rigorous proofs and explanations. The book also includes a number of helpful exercises. Prerequisites for the book include senior undergraduate/beginning graduate-level courses in probability and statistics. This book grew out of lectures delivered at the University of California, Berkeley, over many years. The subject is a part of asymptotics in statistics, organized around a few central ideas. The presentation proceeds from the general to the particular since this seemed the best way to emphasize the basic concepts. The reader is expected to have been exposed to statistical thinking and methodology, as expounded for instance in the book by H. Cramer [1946] or the more recent text by P. Bickel and K. Doksum [1977]. Another possibility, closer to the present in spirit, is Ferguson [1967]. Otherwise the reader is expected to possess some mathematical maturity, but not really a great deal of detailed mathematical knowledge. Very few mathematical objects are used; their assumed properties are simple; the results are almost always immediate consequences of the definitions. Some objects, such as vector lattices, may not have been included in the standard background of a student of statistics. For these we have provided a summary of relevant facts in the Appendix. The basic structures in the whole affair are systems that Blackwell called "experiments" and "transitions" between them. An "experiment" is a mathematical abstraction intended to describe the basic features of an observational process if that process is contemplated in advance of its implementation. Typically, an experiment consists of a set \( E \) of theories about what may happen in the observational process. A broad and unified methodology for robust statistics—with exciting new applications Robust statistics is one of the fastest growing fields in contemporary statistics. It is also one of the more diverse and sometimes confounding areas, given the many different assessments and interpretations of robustness by theoretical and applied statisticians. This innovative book unifies the many varied, yet related, concepts of robust statistics under a sound theoretical modulation. It seamlessly integrates asymptotics and interrelations, and provides statisticians with an effective system for dealing with the interrelations between the various classes of procedures. Drawing on the expertise of researchers from around the world, and covering over a decade’s worth of developments in the field, Robust Statistical Procedures: Asymptotics and Interrelations: Discusses both theory and applications in its two parts, from the fundamentals to robust statistical inference Thoroughly explores the interrelations between diverse classes of procedures, unlike any other book Compares nonparametric procedures with robust statistics, explaining in detail asymptotic representations for various estimators Provides a timesaving list of mathematical tools for the problems under discussion Keeps mathematical abstractions to a minimum, in spite of its largely theoretical content Includes useful problems and exercises at the end of each
chapter Offers strategies for more complex models when using robust statistical procedures Self-contained and rounded in approach, this book is invaluable for both applied statisticians and theoretical researchers; for graduate students in mathematical statistics; and for anyone interested in the influence of this methodology. Our book Asymptotic Techniquesfor Use in Statistics was originally planned as an account of asymptotic statistical theory, but by the time we had completed the mathematical preliminaries it seemed best to publish these separately. The present book, although largely self-contained, takes up the original theme and gives a systematic account of some recent developments in asymptotic parametric inference from a likelihood-based perspective. Chapters 1-4 are relatively elementary and provide first a review of key concepts such as likelihood, sufficiency, conditionality, ancillarity, exponential families and transformation models. Then first-order asymptotic theory is set out, followed by a discussion of the need for higher-order theory. This is then developed in some generality in Chapters 5-8. A final chapter deals briefly with some more specialized issues. The discussion emphasizes concepts and techniques rather than precise mathematical verifications with full attention to regularity conditions and, especially in the less technical chapters, draws quite heavily on illustrative examples. Each chapter ends with outline further results and exercises and with bibliographic notes. Many parts of the field discussed in this book are undergoing rapid further development, and in those parts the book therefore in some respects has more the flavour of a progress report than an exposition of a largely completed theory.

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A broad view of exact statistical inference and the development of asymptotic statistical inference.
Probability and stochastic processes; Limit theorems for some statistics; Asymptotic theory of estimation; Linear parametric inference; Martingale approach to inference; Inference in nonlinear regression; Von mises functionals; Empirical characteristic function and its applications.

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